PART 3 – ANALYSIS AND CALCULATIONS

Offset and scaling circuit analysis and calculation

What follows is an analysis of a circuit for offsetting and scaling a differential output from a sensor, so that it optimally matches the single ended input of a PicoLog 1000 series data logger. The aim of this analysis is to create the equations for calculating the resistor values used on the Small Terminal Board. The first part of this analysis validates the assumptions that we can make about our starting point.

So, from the Small Terminal Board User Guide we have the following circuit diagram of the board:

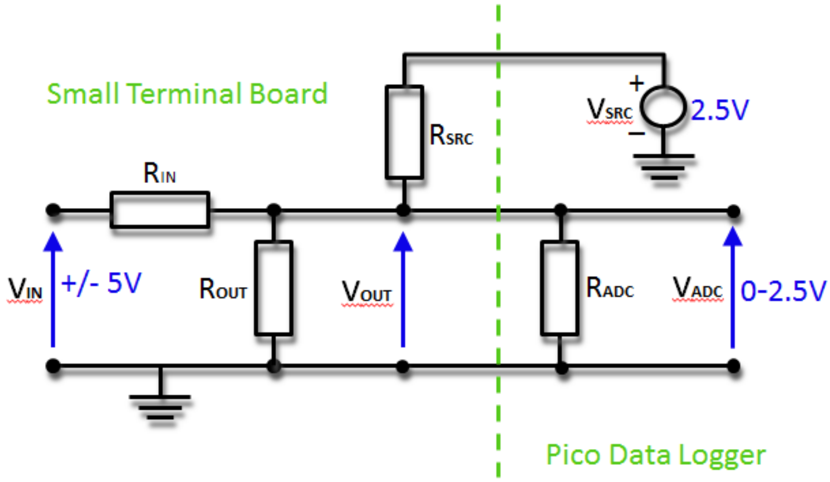

fig 1 --- offset and scaling diagram from Small Terminal Board User Guide

We know that the ADC input impedance R_{ADC} is 1M ohm, which will be large enough to look like an open circuit for the resistor range we will be considering (circumstances where the ADC input impedance is no longer like an open circuit are discussed later in 'Circuit Loading'). So, we can redraw the circuit to show only what we need for the analysis, as follows:

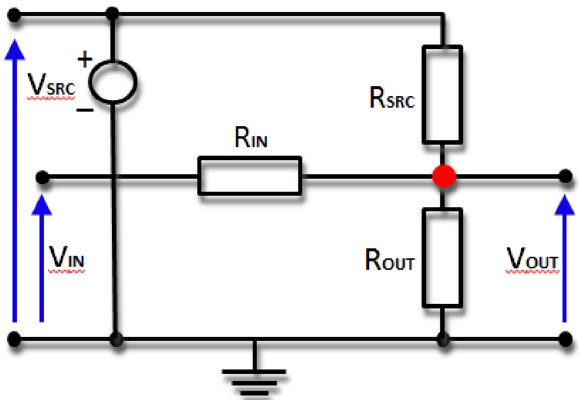

fig 2 --- simplified diagram for analysis

The voltage source (V_{SRC}) is an output from the PicoLog 1012, and PicoLog 1216 and, is independent of the input voltage applied to the terminal board. Looking at the redrawn circuit, there is only one node of significance (in red). So, in order to derive the relationships between the resistors from simple intuitive circuit analysis we can use Kirchoff's Current Law (KCL), which states that the sum of the currents entering a node is equal to the sum of the currents leaving a node. From the redrawn circuit of fig 2 there are only 2 possible scenarios for KCL (shown as follows in fig 3 and fig 4).

In fig 3 the input voltage V_{IN} is larger than the output voltage V_{OUT}, and the current I_{IN} must therefore flow into the node (shown in red). In fig 4 V_{IN} is smaller than V_{OUT}, so I_{IN} must flow out of the node. These 2 scenarios are now considered for equivalence side by side.

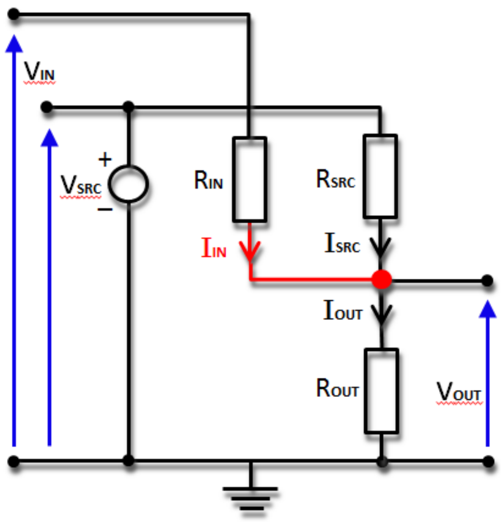

fig 3 --- $V_{IN} > V_{OUT}$

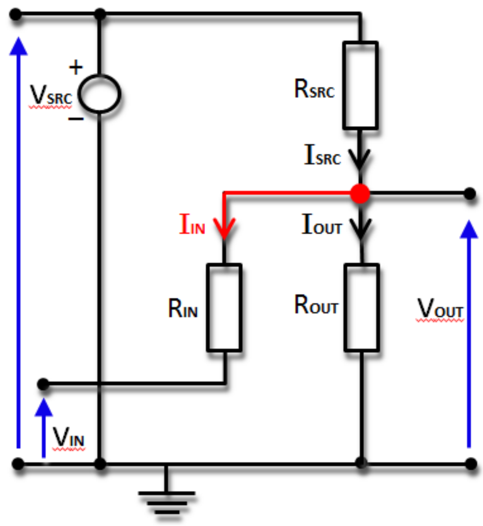

fig 4 --- $V_{OUT} > V_{IN}$

Applying KCL to fig 3 we have:

$$I_{OUT} = I_{SRC} + I_{IN} \qquad \text{Eq [1]}$$

So,

$$\frac{V_{OUT}}{R_{OUT}} = \frac{V_{SRC} - V_{OUT}}{R_{SRC}} + \frac{V_{IN} - V_{OUT}}{R_{IN}} \qquad \text{Eq [2]}$$

$$= \frac{V_{SRC}}{R_{SRC}} - \frac{V_{OUT}}{R_{SRC}} + \frac{V_{IN}}{R_{IN}} - \frac{V_{OUT}}{R_{IN}} \qquad \text{Eq [3]}$$

Applying KCL to fig 4 we have;

$$I_{SRC} = I_{IN} + I_{OUT}$$

So,

$$I_{OUT} = I_{SRC} - I_{IN} \qquad \text{Eq [4]}$$

And,

$$\frac{V_{OUT}}{R_{OUT}} = \frac{V_{SRC} - V_{OUT}}{R_{SRC}} - \frac{V_{OUT} - V_{IN}}{R_{IN}}$$

$$= \frac{V_{SRC}}{R_{SRC}} - \frac{V_{OUT}}{R_{SRC}} - \frac{V_{OUT}}{R_{IN}} + \frac{V_{IN}}{R_{IN}} \qquad \text{Eq [5]}$$

You will notice that Eq [3] is the same as Eq [5], so either circuit representation, with the current flowing into, or out of the node will produce the same results from analysis (it's convenient to use both).
From this point forward we can derive the same resistor ratios from 2 methods of analysis, to cater to the user that prefers either a mathematical based proof, or a more practical explanation based upon circuit behaviour.

Deriving equations through circuit manipulation

Now, for fig 4, when V_{IN} is positive we have;

$$I_{IN} = \frac{V_{OUT} - V_{IN}}{R_{IN}}$$

Note that when V_{IN} is negative this can be expressed using the absolute value $|V_{IN}|$ as follows;

$$I_{IN} = \frac{V_{OUT} - (-|V_{IN}|)}{R_{IN}}$$

So,

$$= \frac{V_{OUT} + |V_{IN}|}{R_{IN}} \qquad \text{Eq [6]}$$

This is easier to visualize, by redrawing the circuit from fig 4 again as follows (the red arrow shows the magnitude):

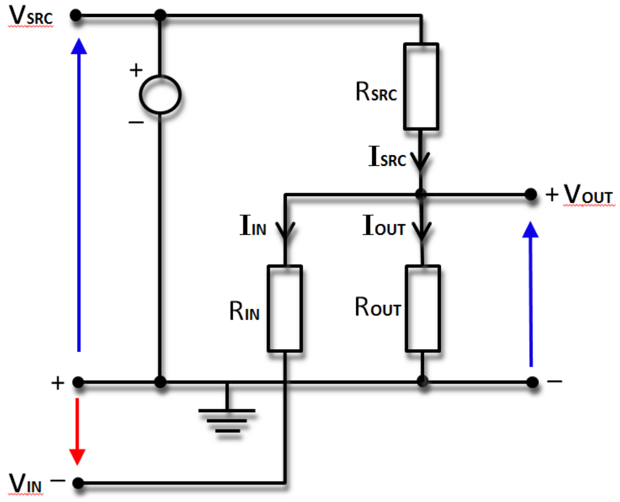

fig 5 --- $V_{IN} < 0V$

From fig 5 the total voltage across R_{IN} is the difference in potential between the V_{OUT} terminal and ground, added to the absolute difference in potential between the V_{IN} terminal and ground (which is the description of Eq [6]).

Re-arranging Eq [6], we have:

$$|V_{IN}| = (I_{IN} \times R_{IN}) - V_{OUT}$$

So, as V_{OUT} becomes smaller, the magnitude of V_{IN} becomes larger, i.e. V_{IN} becomes more negative. When V_{OUT} is at its smallest, i.e. zero (matching the minimum of the input range for the data logger input), there is no current in resistor R_{OUT}, and V_{IN} is at its most negative value $V_{IN(MIN)}$.

So, we can redraw a simpler equivalent circuit, using the circuit when V_{IN} is negative (i.e. fig 5) as follows:

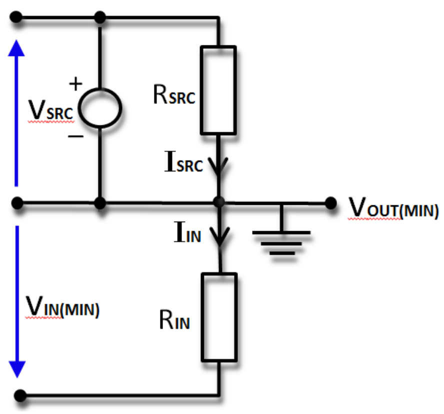

fig 6 --- equivalent circuit for minimum values

Clearly, from fig 6;

$$I_{SRC} = I_{IN}$$

So, from equations [1] & [3] we have;

$$\frac{V_{SRC}}{R_{SRC}} - \frac{V_{OUT(MIN)}}{R_{SRC}} = \frac{V_{OUT(MIN)}}{R_{IN}} - \frac{V_{IN(MIN)}}{R_{IN}}$$

And, as $V_{OUT(MIN)} = 0V$ we have:

$$\frac{V_{SRC}}{R_{SRC}} = - \frac{V_{IN(MIN)}}{R_{IN}}$$

Substituting the actual value of 2.5V for the onboard voltage source V_{SRC} we have:

$$R_{IN} = \left(\frac{-V_{IN(MIN)}}{2.5}\right) R_{SRC} \qquad \text{Eq [7]}$$

(Note that the objective of using the Small Terminal Board with the voltage source is to offset a potentially negative going output voltage to a positive only output voltage, i.e. change a differential output to a single ended output, this means that the minimum input voltage $V_{IN(MIN)}$ will always be negative, and the resistor relationship in Eq [7] will always be positive.)

Now, considering the other extreme (looking at fig 3), when the voltage V_{OUT} is at its maximum it should be matching the maximum of the input range for the data logger, i.e. 2.5V, and the input voltage will be at its maximum. V_{OUT} will then also be at the same potential as the voltage source V_{SRC}, so there will be no current flowing in R_{SRC}. We can therefore redraw a simpler equivalent circuit from fig 3 as follows:

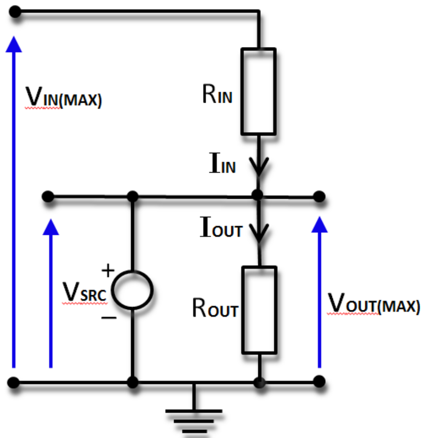

fig 7 --- equivalent circuit for maximum values

Clearly, from fig 7:

$$I_{OUT} = I_{IN}$$

So, from equations [1] & [2] we have;

$$\frac{V_{OUT(MAX)}}{R_{OUT}} = \frac{V_{IN(MAX)} - V_{OUT(MAX)}}{R_{IN}}$$

And, $$R_{OUT} = R_{IN} \left(\frac{V_{OUT(MAX)}}{V_{IN(MAX)} - V_{OUT(MAX)}} \right)$$

Substituting the actual value of 2.5V for V_{OUT} we have:

$$R_{OUT} = \left(\frac{2.5}{V_{IN(MAX)} - 2.5} \right) R_{IN} \qquad \text{Eq [8]}$$

Deriving equations through mathematic equation manipulation

From Eq [3], or Eq [5]:

$$\frac{V_{OUT}}{R_{IN}} + \frac{V_{OUT}}{R_{OUT}} + \frac{V_{OUT}}{R_{SRC}} = \frac{V_{IN}}{R_{IN}} + \frac{V_{SRC}}{R_{SRC}}$$

And, $$V_{OUT} \left(\frac{1}{R_{IN}} + \frac{1}{R_{OUT}} + \frac{1}{R_{SRC}} \right) = V_{IN} \left(\frac{1}{R_{IN}} \right) + V_{SRC} \left(\frac{1}{R_{SRC}} \right)$$

So, $$V_{OUT} = V_{IN} \left(\frac{1/R_{IN}}{1/R_{IN} + 1/R_{OUT} + 1/R_{SRC}} \right) + V_{SRC} \left(\frac{1/R_{SRC}}{1/R_{IN} + 1/R_{OUT} + 1/R_{SRC}} \right) \qquad \text{Eq [9]}$$

Now, using actual values:

The onboard voltage source, V_{SRC} is 2.5V

The limits of the voltage range that the PicoLog inputs can convert are 0V to 2.5V. So, in order to use the maximum range of the ADC in the PicoLog 1000, the interface board must produce outputs of:

$V_{OUT(MIN)}$ = 0V, and $V_{OUT(MAX)}$ = 2.5V

So, simplifying Eq [9] we have:

$$V_{OUT} = V_{IN} (X) + V_{SRC} (Y) \qquad \text{Eq [10]}$$

Now, substituting the minimum values in Eq [10] we have:

$$0 = V_{IN(MIN)} (X) + 2.5(Y)$$

$$\therefore \quad 2.5(Y) = - V_{IN(MIN)} (X)$$

$$\therefore \quad \frac{Y}{X} = - \frac{V_{IN(MIN)}}{2.5}$$

Substituting for X and Y from Eq [9] & [10], and multiplying top and bottom by ($1/R_{IN} + 1/R_{OUT} + 1/R_{SRC}$), we have:

$$\frac{1/R_{SRC}}{1/R_{IN}} = - \frac{V_{IN(MIN)}}{2.5}$$

$$\therefore \quad \frac{R_{IN}}{R_{SRC}} = - \frac{V_{IN(MIN)}}{2.5}$$

$$\therefore \quad R_{IN} = -\left(\frac{V_{IN(MIN)}}{2.5}\right) R_{SRC} \qquad \text{Eq [7]}$$

Substituting the maximum values in Eq [10] we have:

$$2.5 = V_{IN(MAX)}\,(X) + 2.5\,(Y)$$

$$\therefore \quad 2.5 - 2.5\,(Y) = V_{IN(MAX)}\,(X)$$

$$\therefore \quad \frac{1 - Y}{X} = \frac{V_{IN(MAX)}}{2.5}$$

Again, substituting for X and Y from Eq [9] & [10], and multiplying top and bottom by ($1/R_{IN} + 1/R_{OUT} + 1/R_{SRC}$), we have:

$$\frac{(1/R_{IN} + 1/R_{OUT} + 1/R_{SRC}) - 1/R_{SRC}}{1/R_{IN}} = \frac{V_{IN(MAX)}}{2.5}$$

$$\therefore \quad \frac{1/R_{IN} + 1/R_{OUT}}{1/R_{IN}} = \frac{V_{IN(MAX)}}{2.5}$$

Multiplying top and bottom by R_{IN}, we have:

$$1 + \frac{R_{IN}}{R_{OUT}} = \frac{V_{IN(MAX)}}{2.5}$$

$$\therefore \quad \frac{R_{IN}}{R_{OUT}} = \frac{V_{IN(MAX)}}{2.5} - 1$$

$$\therefore \quad R_{OUT} = \left(\frac{2.5}{V_{IN(MAX)} - 2.5}\right) R_{IN} \qquad \text{Eq [8]}$$

Eq [7] & [8] are then, clearly, the same equations as those derived from the analysis through circuit manipulation

Using the Equations

Now, the ideal situation would be to have an expression for each resistor so that known values can be substituted to give the actual resistor values. However, analysis of the circuit across the minimum and maximum range of both input voltage and output voltage provided only 2 scenarios with equations where the unknown quantities could be reduced to just the resistors. The mathematical analysis produced only 2 equations for 3 unknown values to solve simultaneously. So the analysis can only produce expressions in terms of resistor ratios, rather than individual resistor expressions.
However, if we just choose a resistor value we can then use the ratios to calculate the other 2 values we need (although we need to be aware of what the practical limits are for the range of resistor values that we can choose).

So, as an example of how this works, now that we have expressions for the ratio of R_{IN} to R_{SRC} and the ratio of R_{OUT} to R_{IN}, we can insert actual values for $V_{IN(MIN)}$ and $V_{IN(MAX)}$.

From the Small Terminal Board User Guide, the example given considers a voltage range for the sensor output of +/-5V. So, the interface board will receive inputs of:

$V_{IN(MIN)} = -5V$, $V_{IN(MAX)} = +5V$

Inserting these into the resistor ratio equations [7] & [8] we have;

$$R_{IN} = \left(\frac{-(-5)}{2.5} \right) R_{SRC}$$

So, $R_{IN} = 2\ R_{SRC}$

And, $$R_{OUT} = \frac{2.5}{(5 - 2.5)} R_{IN}$$

So, $R_{OUT} = R_{IN}$

So, for instance, if we just chose R_{SRC} to be a value of 10KΩ then, from the equations immediately above for R_{IN} and R_{OUT}, their values will be 20KΩ (which matches the values recommended in the User Guide for a +/-5 volt sensor output range).

Note that for the special case where V_{IN} is +/-2.5V, there is no scaling (resistor potential divider) as the maximum output level is already matched from sensor output to data logger input. So, the circuit is just matching a -2.5V sensor output to a 0V data logger input, i.e. providing a bias (offset). So, from Eq [8] $R_{OUT} = \infty$ i.e. R_{OUT} is replaced by an open circuit (the bottom half of the voltage divider doesn't exist). And, from Eq [7] $R_{IN} = R_{SRC}$, so V_{OUT} is always half way between V_{SRC} and V_{IN} , starting from the minimum value of fig 6, except at the maximum value where $V_{SRC} = V_{OUT}$.

As mentioned previously, before choosing values we need to establish the limits of the resistor values that we can select, based upon the objective of the Interface Circuit. The objective is to match the operating range of the sensor to the input range of the data logger. This needs to be done without reducing the dynamic range, sensitivity, or accuracy of the data acquisition system, by virtue of having added the circuit. The following sections discuss these topics.